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13. ABSTRACT (Maximum 200 words)

Efficient and accurate gridless methods were developed for the simulation of the nonlinear response of solids. Such methods are of potential usefulness in penetration mechanics because they facilitate the modeling of phenomena which involves the creation of new surfaces, such as penetration and fracture, and problems involving high gradients, such as shear bands. Two approaches, moving least mean square interpolants and kernel functions similar to smooth particle hydrodynamics (SPH), have been explored. A correction function was developed and convergence of the corrected approximation was proven for linear problems. Several different approaches were also taken to extending these methods to problems involving large deformations of solids. These methods have been applied to problems involving shear banding and moving cracks. Computations were compared to the Kalthoff experiments and good agreement was achieved with experimental fracture paths. These studies entailed the development of contact-impact algorithms, but within the framework of methodologies based on moving least squares and kernel function interpolants.

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Gridless Computational Methods for Penetration Mechanics

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Summary

The objective of this research is to develop efficient and accurate gridless methods for large deformation problems. Two effective paths to achieving this objective are:

1. the development of the discrete equations through direct application of least mean square interpolants;

2. the improvement of SPH methods by the inclusion of a correction function in the weighting function.

Major issues have been resolved in the extension of two approaches, namely Element Free Galerkin (EFG) and Reproducing Kernel Particle Method (RKPM) from linear analysis to nonlinear large deformation problems by the inclusion of accurate quadrature of the weak form. This appears to be essential to exploiting the full potential accuracy of the methods. We have demonstrated large deformation analysis with two dimensional problems, and problems involving shear banding and moving cracks. Our results compare well with experiments of Kalthoff, which provide an excellent benchmark for these computations since these problems involve both curved cracks and dynamic fracture. We believe that methods which are able to predict the complex phenomena exhibited in the Kalthoff experiments should provide a more sound basis for numerical prediction of penetration phenomena.

Because of the unique capability of the developed gridless methods, we believe that these methods are suitable to model penetration and the resulting development of new surfaces, both from the penetration itself and from fracture. Their potential for improving penetration calculations is truly significant.

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